1. Motivation:

Relations can be useful for two reasons:

1. Multi-Target Tracking, e.g.,
   - two objects moving together,
   - two players in the same team
   
2. Activity Recognition tasks, e.g.,
   - a "waving-hand" to say "Hello" vs a "waving-hand" to call a taxi

2. Basic definitions:

   - **Relational Domain**: the set of objects and relations between them.
   - **Relational State**: the set of the attribute values of the objects in the Domain and their relations.

   \[ S = \begin{pmatrix} s \mid A \\ r \mid A \end{pmatrix} \]

   \( \text{State evolves with time} \)

3. Relational Dynamic BNs:

   - **RDBNs**: pair of RBNs
     - intra-slice distribution -> sensor model;
     - inter-slice distribution -> transition model
   - **RBNs**: set of nodes (one for each attribute and/or relation) whose causality is encoded as a directed graph

4. Inference:

   Under Markov assumption, Bayesian Filter algorithm:

   \[ \text{bel}(s,t) = p(s_t | z_{1:t}) = \sum_{s_{t-1}} p(s_{t-1}, s_t | z_{1:t}) \text{bel}(s_{t-1}) ds_{t-1} \]

5. Assumptions:

   Relations in the State result in correlating the State of different objects between them

   - Sensor model: part of the state relative to relations, \( s' \), not directly observable
     \[ p(z_t | s_t) = p(z_t | s_{t-1}, s'_t) = p(z_t | s'_t) \]
   - Transition model:
     \[ p(s_t | s_{t-1}) = p(s_{t-1}, s'_t | s_{t-1}, s'_t) = p(s_{t-1}, s'_t) \]

   1. Tracking in case of missing values

6. Relational Particle Filter:

   Algorithm 1: Relational Particle Filter algorithm

   \[ X_t = \text{RPF}(X_{t-1}, z_t) \]
   for all \( m = 1 : M \) do
   1. hypothesis for the state of the objects:
      \[ x^{(m)}_{t-1} \sim p(z_t | s_{t-1} = x^{(m)}_{t-1}) \]
   2. hypothesis for the state of the relations:
      \[ x^{(m)}_t = p(s_{t-1} | s_t = x^{(m)}_{t-1}) \]
   3. compute weights:
      \[ \omega^{(m)} = p(z_t | x^{(m)}_{t-1}) \]
   for all \( m = 1 : M \) do
   4. normalize weights:
      \[ \tilde{\omega}^{(m)} = \frac{\omega^{(m)}}{\sum_{m=1}^{M} \omega^{(m)}} \]
   for all \( m = 1 : M \) do
   5. draw \( s \) with probability \( \text{bel}(s^{(m)} | x^{(m)}_{t-1}) \) and add \( X_t^{(m)} \) to the set \( X_t \).

7. Experiments and results:

1) Tracking in case of missing values

2) Tracking in the ships domain